Indefinite Integration

Question1

If $\frac{x^2-3}{(x+2)(x^2+1)}=\frac{A}{x+2}+\frac{Bx+C}{(x^2+1)}$**, then** $3A+2B-C=$

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Options:

A.

$\frac{8}{5}$

B.

$\frac{16}{5}$

C.

$\frac{3}{5}$

D.

$\frac{19}{5}$

Answer: D

Solution:

We are given:

$\frac{x^2-3}{(x+2)(x^2+1)}=\frac{A}{x+2}+\frac{Bx+C}{x^2+1}$

First, get a common denominator to combine the right side:

$\frac{A}{x+2}+\frac{Bx+C}{x^2+1}=\frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$

Set the numerators equal to each other:

$x^2 - 3 = A(x^2 + 1) + (Bx + C)(x + 2)$

Expand the right side:

$A(x^2 + 1) = Ax^2 + A$
$(Bx + C)(x + 2) = Bx^2 + 2Bx + Cx + 2C = Bx^2 + (2B + C)x + 2C$

Combine all terms on the right:

$Ax^2 + Bx^2 = (A + B)x^2$
$(2B + C)x$
$A + 2C$

So we get:

$x^2 - 3 = (A + B)x^2 + (2B + C)x + (A + 2C)$

Now, match the coefficients for x^2, x, and the constant term:

For x^2: $A + B = 1$
For x: $2B + C = 0$
For constant: $A + 2C = -3$

Write the equations:

1. $A + B = 1$
2. $2B + C = 0$
3. $A + 2C = -3$

Now solve these equations step by step:

From equation 2: $2B + C = 0 \rightarrow C = -2B$

Substitute $C = -2B$ into equation 3: $A + 2(-2B) = -3$
$A - 4B = -3$

Now use $A + B = 1$. Substitute $A = 1 - B$ into the last equation:
$1 - B - 4B = -3$
$1 - 5B = -3$
$-5B = -4$
$B = \frac{4}{5}$

Now $C = -2B = -2 \times \frac{4}{5} = \frac{-8}{5}$
$A = 1 - B = 1 - \frac{4}{5} = \frac{1}{5}$

Now find $3A + 2B - C$:

$3\left(\frac{1}{5}\right) + 2\left(\frac{4}{5}\right) - \left(\frac{-8}{5}\right)$
$= \frac{3}{5} + \frac{8}{5} + \frac{8}{5}$
$= \frac{3+8+8}{5}$
$= \frac{19}{5}$

Question2

$$\int\left(\frac{1}{x^2}+\frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}\right)dx=$$

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Options:

A.

$\frac{(\sin x-\cos x)x-\sin x\cos x}{x\sin x\cos x}+C$

B.

$-\frac{1}{x}+\frac{\sin x+\cos x}{\cos x-\sin x}+c$

C.

$-\frac{1}{x}+\frac{\sin x-\cos x}{\sin^2 x\cos^2 x}+C$

D.

$\frac{(\sin x-\cos x)x-\sin x-\cos x}{x(\sin x+\cos x)}+C$

Answer: A

Solution:

Let $I=\int\left(\frac{1}{x^2}+\frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}\right)dx$

again let $I_1=\int\frac{1}{x^2}dx$

And $I_2=\int\frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}dx$

$\therefore I_1=\int x^{-2}dx=\frac{x^{-2+11}}{-2+1}+c_1=-\frac{1}{x}+c_1$

And $I_2=\int\left(\frac{\sin^3 x}{\sin^2 x\cos^2 x}+\frac{\cos^3 x}{\sin^2 x\cos^2 x}\right)dx$

$=\int(\tan x\sec x+\cot x\operatorname{cosec} x)dx$

$=\int\tan x\sec x dx+\int\cot x\operatorname{cosec} x dx$

$=\sec x+(-\operatorname{cosec} x)+c_2$

$\therefore I=I_1+I_2=-\frac{1}{x}+\sec x-\operatorname{cosec} x+C$

$\Rightarrow I=-\frac{1}{x}+\frac{1}{\cos x}-\frac{1}{\sin x}+C$

$= \frac{-(\cos x \sin x)+x \sin x-x \cos x}{x \cos x \sin x} + C$

$= \frac{x(\sin x-\cos x)-\cos x \sin x}{x \cos x \sin x} + C$

Question3

If $I_n = \int \frac{1}{(x^2+1)^n} dx$**, then** $2nI_{n+1} - (2n-1)I_n =$

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Options:

A.

$\frac{(x^2+1)^n}{x} + C$

B.

$\frac{x}{(x^2+1)^n} + C$

C.

$x(x^2+1)^{n-1} + C$

D.

$\frac{x}{(x^2+1)^{n-1}} + C$

Answer: B

Solution:

Given, $I_n = \int \frac{1}{(x^2+1)^n} dx$

$= \frac{x}{(x^2+1)^n} - \int \left(\frac{d}{dx}\left(\frac{1}{(x^2+1)^n}\right) \int dx\right) dx$

$$=\frac{x}{\left(x^2+1\right)^n}+2n\int\frac{x^2}{\left(x^2+1\right)^{n+1}}dx$$

$$\Rightarrow I_n=\frac{x}{\left(x^2+1\right)^n}+2n\int\left(\frac{1}{\left(x^2+1\right)^n}-\frac{1}{\left(x^2+1\right)^{n+1}}\right)dx$$

$$\Rightarrow I_n=\frac{x}{\left(x^2+1\right)^n}+2n\left(I_n-I_{n+1}\right)+C$$

$$\Rightarrow 2nI_{n+1}=\frac{x}{\left(x^2+1\right)^n}+(2n-1)I_n+C$$

$$\Rightarrow 2nI_{n+1}-(2n-1)I_n=\frac{x}{\left(x^2+1\right)^n}+C$$

Question4

$$\int\frac{x^3}{x^4+3x^2+2}dx=$$

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Options:

A.

$\log\left(\frac{x^2+2}{\sqrt{x^2+1}}\right)+C$

B.

$\log\left(x^2+2\right)-2\log\left(x^2+1\right)+C$

C.

$\log\left(\frac{\left(x^2+2\right)x}{\sqrt{x^2+1}}\right)+C$

D.

$\log\left(\frac{x^2+1}{\sqrt{x^2+2}}\right)+C$

Answer: A

Solution:

Let $I=\int \frac{x^3 dx}{x^4+3x^2+2}$

put $x^2=t \Rightarrow 2xdx=dt \Rightarrow xdx=\frac{dt}{2}$

$$\therefore I=\int \frac{t \cdot \frac{dt}{2}}{t^2+3t+2}$$
$$I=\frac{1}{2}\int \frac{tdt}{t^2+3t+2}=\frac{1}{2}\int \frac{tdt}{(t+1)(t+2)}$$

Now, $\frac{t}{(t+1)(t+2)}=\frac{A}{t+1}+\frac{B}{t+2}$

$\Rightarrow \quad t=A(t+2)+B(t+1)$

When, $t+2=0 \Rightarrow t=-2$

$\therefore \quad -2=B(-1) \Rightarrow B=2$

when, $t+1=0 \Rightarrow t=-1$

$$\therefore \quad -1=A(1)+B(0) \Rightarrow A=-1$$
$$\therefore \frac{t}{(t+1)t+2)}=\frac{-1}{t+1}+\frac{2}{t+2}$$
$$\therefore I=\frac{1}{2}\int\left(\frac{-1}{t+1}+\frac{2}{t+2}\right)dt$$

$$I=\frac{1}{2}[-\log(t+1)+2\log(t+2)]+C$$
$$=-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)\Big]+C$$
$$=\log\left(x^2+2\right)+\log\left(x^2+1\right)^{-1/2}+C$$
$$=\log\left(\frac{x^2+2}{\sqrt{x^2+1}}\right)+C$$

Question5

If $\int \frac{dx}{(x^2+9)\sqrt{x^2+16}}=\frac{1}{3\sqrt{7}}\tan^{-1}\left(K\frac{x}{\sqrt{16+x^2}}\right)+c$**, then** $K=$

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Options:

A.

$\frac{\sqrt{7}}{3}$

B.

$3\sqrt{7}$

C.

$\frac{3}{\sqrt{7}}$

D.

$\frac{3}{7}$

Answer: A

Solution:

$$\text{Let } I=\int \frac{dx}{\left(x^2+9\right)\sqrt{x^2+16}}$$

$$\text{put } x=4\tan\theta \Rightarrow dx=4\sec^2\theta d\theta$$

$$\therefore I=\int \frac{4\sec^2\theta d\theta}{\left(16\tan^2\theta+9\right)\sqrt{16\tan^2\theta+16}}$$

$$=\int \frac{4\sec^2\theta d\theta}{\left(16\tan^2\theta+9\right)4\sec\theta}$$

$$=\int \frac{\sec\theta d\theta}{16\tan^2\theta+9}$$

$$=\int \frac{\cos\theta d\theta}{16\sin^2\theta+9\cos^2\theta}=\int \frac{\cos\theta d\theta}{7\sin^2\theta+9}$$

$$\text{Again put } \sin\theta=t$$

$$\Rightarrow \cos\theta d\theta=dt$$

$$\therefore I=\int \frac{dt}{7t^2+9}$$

$$I=\int \frac{dt}{7\left(t^2+\frac{9}{7}\right)}=\frac{1}{7}\int \frac{dt}{t^2+\left(\frac{3}{\sqrt{7}}\right)^2}$$

$$=\frac{1}{7}\cdot\frac{1}{\frac{3}{\sqrt{7}}}\tan^{-1}\left(\frac{t}{\frac{3}{\sqrt{7}}}\right)+C$$

$$=\frac{1}{3\sqrt{7}}\tan^{-1}\left(\frac{\sqrt{7}t}{3}\right)+C$$

$$=\frac{1}{3\sqrt{7}}\tan^{-1}\left(\frac{\sqrt{7}\sin\theta}{3}\right)+C$$

$$=\frac{1}{3\sqrt{7}}\tan^{-1}\left(\frac{\sqrt{7}x}{3\sqrt{x^2+16}}\right)+C$$

$$\left(\because x = 4\tan\theta \Rightarrow \sin\theta = \frac{x}{\sqrt{x^2+16}}\right)$$

$$\therefore \quad K = \frac{\sqrt{7}}{3}$$

Question6

$$\int \frac{2\sin x - 3\cos x}{4\cos x - 3\sin x} dx =$$

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Options:

A.

$\frac{1}{25}[17\log|4\cos x - 3\sin x| - 6x] + C$

B.

$\frac{1}{25}[x - 18\log|4\cos x - 3\sin x|] + C$

C.

$\frac{1}{25}[\log|4\cos x - 3\sin x| - 18x] + C$

D.

$\frac{1}{25}[17x - 6\log|4\cos x - 3\sin x|] + C$

Answer: C

Solution:

$I = \int \frac{2\sin x - 3\cos x}{4\cos x - 3\sin x} dx$

Let $t = 4\cos x - 3\sin x$

$\Rightarrow \frac{dt}{dx} = -4\sin x - 3\cos x$

Now, $2\sin x - 3\cos x$

$$\Rightarrow A(-4\sin x - 3\cos x) + B(4\cos x - 3\sin x)$$
$$\Rightarrow (-4A - 3B)\sin x + (-3A + 4B)\cos x$$

Comparing the coefficients, we get

$-4A - 3B = 2$ and $-3A + 4B = -3$

Solving these two equations, we get

$A = \frac{1}{25}$ and $B = \frac{-18}{25}$

$$\text{So, } I = \int \frac{2\sin x - 3\cos x}{4\cos x - 3\sin x} dx$$
$$\Rightarrow \frac{1}{25} \int \frac{-4\sin x - 3\cos x}{4\cos x - 3\sin x} dx + \left(\frac{-18}{25}\right) \int \frac{4\cos x - 3\sin x}{4\cos x - 3\sin x} dx$$
$$\Rightarrow \frac{1}{25} \int \frac{dt}{t} - \frac{18}{25} \int dx \Rightarrow \frac{1}{25} \log|t| - \frac{18}{25} x + C$$
$$\Rightarrow \frac{1}{25} \log|4\cos x - 3\sin x| - \frac{18}{25} x + C$$
$$= \frac{1}{25} [\log|4\cos x - 3\sin x| - 18x] + C$$

Question7

$$\int e^{4x}(\sin 3x - \cos 3x) dx =$$

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Options:

A.

$\frac{e^{4x}}{25}(7\sin 3x - \cos 3x) + C$

B.

$\frac{e^{4x}}{25}(\sin 3x - 7\cos 3x) + C$

C.

$\frac{e^{4x}}{5}(7\sin 3x + \cos 3x) + C$

D.

$\frac{e^{4x}}{5}(\sin 3x + 7\cos 3x) + C$

Answer: B

Solution:

$$\int e^{4x}(\sin 3x - \cos 3x)dx$$
$$= \int e^{4x}\sin 3x dx - \int e^{4x}\cos 3x dx$$

Using the standard integrals

$$\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$
$$\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

Here, $a = 4, b = 3$

$$\text{So, } \int e^{4x}(\sin 3x - \cos 3x)dx$$
$$= \frac{e^{4x}}{4^2+3^2}(4\sin 3x - 3\cos 3x) - \frac{e^{4x}}{4^2+3^2}(4\cos 3x + 3\sin 3x) + C$$
$$\Rightarrow \frac{e^{4x}}{25}[4\sin 3x - 3\cos 3x - 4\cos 3x - 3\sin 3x] + C$$
$$\Rightarrow \frac{e^{4x}}{25}[\sin 3x - 7\cos 3x] + C$$

Question8

$$\int\left(\frac{1-\log x}{1+(\log x)^2}\right)^2 dx =$$

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Options:

A.

$\frac{1}{1+(\log x)^2} + C$

B.

$\frac{\log x}{1+(\log x)^2} + C$

C.

$\frac{x}{1+(\log x)^2} + C$

D.

$\frac{x^2}{1+(\log x)^2} + C$

Answer: C

Solution:

$I = \int \left(\frac{1-\log x}{1+(\log x)^2}\right)^2 dx$

Let $y = \log x$. Then, $x = e^y \Rightarrow dx = e^y \cdot dy$

So, $I = \int \left(\frac{1-y}{1+y^2}\right)^2 e^y dy$

$$= \int \left(\frac{1+y^2-2y}{(1+y^2)^2}\right) e^y dy$$
$$\Rightarrow \quad \int \left(\frac{1}{1+y^2} - \frac{2y}{(1+y^2)^2}\right) e^y dy$$

Since, $\int (f(y) + f'(y)) e^y dy = f(y) e^y + C$

Here, $f(y) = \frac{1}{1+y^2}$ and $f'(y) = \frac{-2y}{(1+y^2)^2}$

$$\therefore \quad I = f(y) e^y + C = \frac{1}{1+y^2} e^y + C$$
$$= \frac{1}{1+(\log x)^2} \cdot e^{\log x} + C$$
$$= \frac{x}{1+(\log x)^2} + C$$

Question9

If $\int (x+2)\sqrt{x^2 - x + 2}\,dx = \frac{1}{3} f(x) + \frac{5}{8} g(x) + \frac{35}{16} h(x) + C$ then $f(-1) + g(-1) + h\left(\frac{1}{2}\right) =$

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Options:

A.

-4

B.

$2+\ln\left(\frac{\sqrt{7}}{2}\right)$

C.

4

D.

-2

Answer: B

Solution:

$$\text{Given, } \int(x+2)\sqrt{x^2-x+2dx}$$

$$\Rightarrow \quad \frac{1}{3}f(x)+\frac{5}{8}g(x)+\frac{35}{16}h(x)+C$$

Let $x+2=A(2x-1)+B$

Comparing coefficients, we get

$$2A=1$$

$$\Rightarrow \quad A=\frac{1}{2}, -A+B=2$$

$$\Rightarrow \quad B=2+A=2+\frac{1}{2}=\frac{5}{2}$$

$$\text{So, } x+2=\frac{1}{2}(2x-1)+\frac{5}{2}$$

$$\int(x+2)\sqrt{x^2-x+2dx}$$

$$=\int\left(\frac{1}{2}(2x-1)+\frac{5}{2}\right)\sqrt{x^2-x+2dx}$$

$$=\frac{1}{2}\int(2x-1)\sqrt{x^2-x+2dx}+\frac{5}{2}\int\sqrt{x^2-x+2dx}$$

Let $u=x^2-x+2$, then $du=(2x-1)dx$

$$=\frac{1}{2}\int\sqrt{u}du+\frac{5}{2}\int\sqrt{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}dx$$

$$=\frac{1}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}}+\frac{5}{2}\left[\frac{2x-1}{4}\sqrt{x^2-x+2}+\frac{7}{8}\ln\left|x-\frac{1}{2}+\sqrt{x^2-x+2}\right|\right]+c$$

$$=\frac{1}{3}\left(x^2-x+2\right)^{\frac{3}{2}}+\frac{5(2x-1)}{8}\sqrt{x^2-x+2}+\frac{35}{16}\ln\left|x-\frac{1}{2}+\sqrt{x^2-x+2}\right|+c$$

Comparing with original equation, we get

$$f(x)=\left(x^2-x+2\right)^{\frac{3}{2}}$$
$$g(x)=(2x-1)\sqrt{x^2-x+2}$$
$$h(x)=\ln\left|x-\frac{1}{2}+\sqrt{x^2-x+2}\right|$$

So, $f(-1)=\left[(-1)^2-(-1)+2\right]^{\frac{3}{2}}$

$$=[1+1+2]^{\frac{3}{2}}=4^{\frac{3}{2}}=8$$
$$g(-1)=(2(-1)-1)\sqrt{(-1)^2-(-1)+2}$$
$$=-3\sqrt{1+1+2}=-3\sqrt{4}$$
$$=-3\times 2=-6$$
$$h\left(\frac{1}{2}\right)=\ln\left|\frac{1}{2}-\frac{1}{2}+\sqrt{\left(\frac{1}{2}\right)^2-\frac{1}{2}+2}\right|$$
$$\Rightarrow \ln\left|0+\sqrt{\frac{1}{4}-\frac{1}{2}+2}\right|$$
$$\Rightarrow \ln\left|\sqrt{\frac{1-2+8}{4}}\right| \Rightarrow \ln\left|\sqrt{\frac{7}{4}}\right|=\ln\left(\frac{\sqrt{7}}{2}\right)$$

So, $f(-1)+g(-1)+h\left(\frac{1}{2}\right)=8-6+\ln\left(\frac{\sqrt{7}}{2}\right)$

$$=2+\ln\left(\frac{\sqrt{7}}{2}\right)$$

Question10

$$\int \frac{\sec x}{3(\sec x+\tan x)+2}\,dx=$$

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Options:

A. $\frac{1}{2}\log\left|\frac{\tan\frac{x}{2}+1}{\tan\frac{x}{2}+5}\right|+C$

B. $\frac{2}{\sqrt{11}}\tan^{-1}\left\{\frac{3\tan\frac{x}{2}+4}{\sqrt{11}}\right\}+C$

C. $\log|3\sec x+2\tan x|+C$

D. $\log|3\tan x+2\sec x|+C.$

Answer: A

Solution:

Let $u=\tan\left(\frac{x}{2}\right) \Rightarrow du=\frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx$

$\Rightarrow$ Using the substitution $\sin x=\frac{2u}{u^2+1}$

and $\cos x=\frac{1-u^2}{u^2+1}$ and $dx=2\frac{du}{u^2+1}$

$\Rightarrow \int \frac{2du}{(1-u^2)\left[3\left(\frac{u^2+1}{1-u^2}-\frac{2u}{u^2-1}\right)+2\right]}$

then we get;

$\int \frac{2}{u^2+6u+5}du=2\int \frac{1}{(u+3)^2-4}du$

Substitute $s=u+3$ and $ds=du$

$=2\int \frac{1}{s^2-4}ds=2\int \frac{-1}{4\left(1-\frac{s^2}{4}\right)}$

$=\frac{-1}{2}\int \frac{1}{1-\frac{s^2}{4}}ds \Rightarrow p=s/2$

$dp=\frac{ds}{2}=-\int \frac{1}{1-p^2}dp=-\tan h^{-1}(p)+C$

$\Rightarrow -\tan h^{-1}\left(\frac{u+3}{2}\right)+C$

$\Rightarrow -\tan h^{-1}\left[\left(\frac{1}{2}\left(\tan\left(\frac{x}{2}\right)+3\right)\right)\right]+C$

$=\frac{1}{2}\left[\log\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right]-\log\left[\sin\left(\frac{x}{2}\right)+5\cos\left(\frac{x}{2}\right)\right]+C$

$=\frac{1}{2}\log\left[\frac{\sin x/2+\cos x/2}{\sin x/+5\cos x/2}\right]+C$

$=\frac{1}{2}\log\left[\frac{\tan x/2+1}{\tan x/2+5}\right]+C$

Question11

$\int \frac{dx}{4+3\cot x}dx=$

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Options:

A. $-\frac{3}{25}\log|4+3\cot x|+\frac{4}{25}x+c$

B. $-\frac{3}{25}\log|4\sin x+3\cos x|+\frac{4}{25}x+c$

C. $\frac{4}{25}\log|4\sin x+3\cos x|-\frac{3}{25}x+c$

D. $\frac{4}{25}\log|4+3\cot x|-\frac{3}{25}x+c$

Answer: B

Solution:

$I=\int\frac{dx}{4+3\cot x}=\int\frac{\sin x}{4\sin x+3\cos x}dx$

$\sin x=1(4\sin x+3\cos x)+m(4\cos x-3\sin x)$

$1=4l-3m$

$0=3l+4m \quad 4l-3\left(-\frac{3}{4}l\right)=1$

$4m=-3l \quad 4l+\frac{9l}{4}=1$

$m=-\frac{3}{4}l \quad \frac{25}{4}l=1$

$l=\frac{4}{25}$

$m=-\frac{3}{4}\times\frac{4}{25}=-\frac{3}{25}$

$$I=\frac{4}{25}\int dx+\left(-\frac{3}{25}\right)\int\frac{4\cos x-3\sin x}{4\sin x+3\cos x}dx$$

$$=\frac{4x}{25}-\frac{3}{25}\ln(4\sin x+3\cos x)+C$$

Question12

$$\int\frac{dx}{(x+1)\sqrt{x^2+4}}=$$

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Options:

A. $\frac{1}{2}\sqrt{\frac{x+1}{x+2}}+c$

B. $\log\left|\frac{x+2}{x+1}\right|+c$

C. $-\frac{1}{\sqrt{5}}\sinh^{-1}\left(\frac{4-x}{2(x+1)}\right)+c$

D. $-\frac{1}{\sqrt{5}}\cosh^{-1}\left(\frac{4+x}{2(x-1)}\right)+c$

Answer: C

Solution:

$\int \frac{dx}{(x+1)\sqrt{x^2+4}}$

Put, $x+1=\frac{1}{t} \Rightarrow dx=-\frac{1}{t^2}dt$

$=\int \frac{-dt}{\sqrt{1-2t+5t^2}}$

$=\int \frac{-\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{\left(\frac{1}{t}-1\right)^2+4}}$

$=\int \frac{-\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{\frac{1}{t^2}+1-\frac{2}{t}+4}}$

$=-\frac{1}{\sqrt{5}}\int \frac{dt}{\sqrt{t^2-\frac{2t}{5}+\frac{1}{5}}}$

$=-\frac{1}{\sqrt{5}}\int \frac{dt}{\sqrt{t^2-\frac{2t}{5}+\frac{1}{5}+\frac{1}{25}-\frac{1}{25}}}$

$=-\frac{1}{\sqrt{5}}\int \frac{dt}{\sqrt{\left(t-\frac{1}{5}\right)^2+\frac{4}{25}}}$

$=-\frac{1}{\sqrt{5}}\int \frac{dt}{\sqrt{\left(t-\frac{1}{5}\right)^2+\left(\frac{2}{5}\right)^2}}$

$=-\frac{1}{\sqrt{5}}\sinh^{-1}\left(\frac{t-(1/5)}{2/5}\right)$

$=-\frac{1}{\sqrt{5}}\sinh^{-1}\left(\frac{5t-1}{2}\right)$

$=-\frac{1}{\sqrt{5}}\sinh^{-1}\left(\frac{\frac{5}{x+1}-1}{2}\right)$

$=-\frac{1}{\sqrt{5}}\sinh^{-1}\left(\frac{5-x-1}{2(x+1)}\right)$

$=-\frac{1}{\sqrt{5}}\sinh^{-1}\left(\frac{4-x}{2(x+1)}\right)+c$

Question13

If $\int e^x\left(x^3+x^2-x+4\right)dx=e^x f(x)+c$, then $f(1)=$

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Options:

A. 0

B. 1

C. 2

D. 3

Answer: D

Solution:

$\int e^x \left(x^3 + x^2 - x + 4\right) dx$

Column I	Column II
$+\left(x^3 + x^2 - x + 4\right)$	e^x
$-\left(3x^2 + 2x - 1\right)$	e^x
$+(6x + 2)$	e^x
-6	e^x
0	e^x

$= e^x \left[x^3 + x^2 - x + 4 - 3x^2 - 2x + 1 + 6x + 2 - 6\right] + C$

$= e^x \left[x^3 - 2x^2 + 3x + 1\right] + C$

$\therefore f(x) = x^3 - 2x^2 + 3x + 1$

$f(1) = 1 - 2 + 3 + 1 = 5 - 2 = 3$

Question14

If $\frac{1}{x^4+x^2+1} = \frac{Ax+B}{x^2+ax+1} + \frac{Cx+D}{x^2-ax+1}$, then $A + B - C + D =$

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Options:

A. a

B. $2a$

C. $3a$

D. $4a$

Answer: B

Solution:

We know that

$x^4+x^2+1=\left(x^2+x+1\right)\left(x^2-x+1\right)$

$\{\therefore a=1\}$

$\frac{1}{x^4+x^2+1}=\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{x^2-x+1}$

$\frac{1}{\left(x^2+x+1\right)\left(x^2-x+1\right)}$

$=\frac{(Ax+B)\left(x^2-x+1\right)+(Cx+D)\left(x^2+x+1\right)}{\left(x^2+x+1\right)\left(x^2-x+1\right)}$

$1=(Ax+B)\left(x^2-x+1\right)+(Cx+D)\left(x^2+x+1\right)$

$1=Ax^3-Ax^2+Ax+Bx^2-Bx+B$
$\quad +Cx^3+Cx^2+Cx+Dx^2+Dx+D$

On comparing, we get

Coefficient of $x^3=0A+C=0$

Coefficient of $x^2=0$

$-A+B+C+D=0$

Coefficient of $x=0$

$A-B+C=0$
$B+D=1$

From Eqs. (i) and (iv), we get

$A-C=1$

and from Eqs. (i) and (v) we get,

$A=\frac{1}{2}, C=-\frac{1}{2}$

So, $A+B-C+D=1=2a$

Question15

If $\int \frac{1}{x^4+8x^2+9} dx=\frac{1}{k}\left[\frac{1}{\sqrt{14}} \tan^{-1}(f(x))-\frac{1}{\sqrt{2}} \tan^{-1}(g(x))\right]+c$ **then,**

$\sqrt{\frac{k}{2}+f(\sqrt{3})+g(1)}=$

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Options:

A. $3-2\sqrt{2}$

B. $\sqrt{2}-1$

C. $\sqrt{3}+2\sqrt{2}$

D. $\sqrt{2}+1$

Answer: A

Solution:

$\int \frac{dx}{x^4+8x^2+9}=\int \frac{dx}{\left(x^2+4-\sqrt{7}\right)\left(x^2+4+\right.}$

Use partial fraction,

$\Rightarrow \int \frac{dx}{\left(x^2+4-\sqrt{7}\right)\left(x^2+4+\sqrt{7}\right)}=\frac{1}{2\sqrt{7}}$

$\int \frac{\left(x^2+4+\sqrt{7}\right)-\left(x^2+4-\sqrt{7}\right)}{\left(x^2+4-\sqrt{7}\right)\left(x^2+4+\sqrt{7}\right)}dx$

$=\frac{1}{2\sqrt{7}}\left[\int \frac{dx}{x^2+4-\sqrt{7}}-\int \frac{dx}{x^2+4+\sqrt{7}}\right]$

$=\frac{1}{2\sqrt{7}}\left[\frac{1}{\sqrt{4-\sqrt{7}}}\tan^{-1}\left(\frac{x}{\sqrt{4-\sqrt{7}}}\right)\right.$

$\left.-\frac{1}{\sqrt{4+\sqrt{7}}}\tan^{-1}\left(\frac{x}{\sqrt{4+\sqrt{7}}}\right)\right]+C$

There is no comparison hence, no options are correct.

Question16

If $\int\left(1+x-x^{-1}\right)e^{\left(x+x^{-1}\right)}dx=f(x)+C$**, then** $f(1)-f(-1)=$

Options:

A. $e^2 - \frac{1}{e^2}$

B. $e^2 + \frac{1}{e^2}$

C. $e + \frac{1}{e}$

D. $e - \frac{1}{e}$

Answer: B

Solution:

To find $f(1) - f(-1)$, we start by evaluating the given integral:

$\int \left(1 + x - \frac{1}{x}\right) e^{x+\frac{1}{x}} \, dx$

This can be expanded as:

$\int e^{x+\frac{1}{x}} \, dx + \int x \left(1 - \frac{1}{x^2}\right) e^{x+\frac{1}{x}} \, dx$

Integrating by parts, we have:

$= xe^{x+\frac{1}{x}} - \int x \left(1 - \frac{1}{x^2}\right) e^{x+\frac{1}{x}} \, dx + \int x \left(1 - \frac{1}{x}\right) e^{x+\frac{1}{x}} \, dx$

This simplifies to:

$= xe^{x+\frac{1}{x}} + C$

Thus, the function $f(x)$ is given by:

$f(x) = xe^{x+\frac{1}{x}}$

To find $f(1)$ and $f(-1)$:

$f(1) = 1 \cdot e^{1+\frac{1}{1}} = e^2$

$f(-1) = -1 \cdot e^{-1+\frac{1}{-1}} = -1 \cdot e^{-2} = -e^{-2}$

Therefore, the difference is:

$f(1) - f(-1) = e^2 - (-e^{-2}) = e^2 + e^{-2}$

Question17

$\int \frac{1}{x^m \sqrt[m]{x^m+1}} dx =$

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Options:

A. $\frac{1}{m-1}\left(\frac{\sqrt[m]{x^m+1}}{x}\right)^m + C$

B. $\frac{-1}{m-1}\left(\frac{\sqrt[m]{x^m+1}}{x}\right)^{m-1} + C$

C. $\frac{-1}{m}\left(\frac{\sqrt[m]{x^m+1}}{x}\right)^m + C$

D. $\frac{1}{m}\left(\frac{\sqrt[m-1]{x^m+1}}{x}\right)^m + C$

Answer: B

Solution:

To evaluate the integral $\int \frac{1}{x^m \sqrt[m]{x^m+1}} dx$, we can use the substitution method.

First, rewrite the integral as:

$\int \frac{1}{x^{m+1} \sqrt[m]{1+\frac{1}{x^m}}} dx$

Now, let:

$1 + \frac{1}{x^m} = t$

Then, differentiate both sides with respect to x:

$-\frac{m}{x^{m+1}} dx = dt$

This implies:

$\frac{dx}{x^{m+1}} = -\frac{1}{m} dt$

Substituting in the integral gives:

$\int -\frac{1}{m} t^{-\frac{1}{m}} dt$

Integrating with respect to t:

$-\frac{1}{m} \cdot \frac{t^{-\frac{1}{m}+1}}{-\frac{1}{m}+1} + C$

Simplify the expression:

$-\frac{1}{m-1}\left(t^{\frac{m-1}{m}}\right)+C$

Substitute back the expression for t:

$-\frac{1}{m-1}\left(\frac{1+x^m}{x^m}\right)^{\frac{m-1}{m}}+C$

Hence, the solution is:

$-\frac{1}{m-1}\left(\frac{\sqrt[m]{x^m+1}}{x}\right)^{m-1}+C$

Question18

If $\int(\sqrt{\operatorname{cosec} x+1})dx=k\tan^{-1}(f(x))+C$, then $\frac{1}{k}f\left(\frac{\pi}{6}\right)=$

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Options:

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $-\frac{1}{4}$

D. $-\frac{1}{2}$

Answer: A

Solution:

$\int\sqrt{(\operatorname{cosec} x+1)}dx=k\tan^{-1}(f(x))+c$

$\Rightarrow\int\sqrt{\frac{1+\sin x}{\sin x}}dx$

$\int\sqrt{\frac{(1+\sin x)(1-\sin x)}{\sin x(1-\sin x)}}dx$

$\int\sqrt{\frac{1-\sin^2 x}{\sin x(1-\sin x)}}dx$

$\int\frac{\cos x dx}{\sqrt{\sin x(1-\sin x)}}$

Let $\sin x=t$

$\cos x dx = dt$

$\Rightarrow \quad \int \frac{dt}{\sqrt{t-t^2}}$

$\int \frac{dt}{\sqrt{\frac{1}{4}-\left(\frac{1}{2}-t\right)^2}}$

Let $\frac{1}{2} - t = u \Rightarrow -dt = du$

then, $2 \int \frac{-du}{\sqrt{1-(2\mu)^2}}$

$= -2\frac{1}{2}\left(\sin^{-1}(4t)\right) + C$

$= -\sin^{-1}(1-2t) + C$

$= -\sin^{-1}(1 - 2\sin x) + C$

$= -\tan^{-1}\left[\frac{1-2\sin x}{2\sqrt{\sin^2 x - \sin x}}\right] + C$

On comparing, we get

$k = -1$ and $f(x) = \frac{1-2\sin x}{2\sqrt{\sin^2 x - \sin x}}$

$kf\left(\frac{\pi}{6}\right) = -1 \times \left[\frac{1-2\times\frac{1}{2}}{2\sqrt{\frac{1}{4}-\frac{1}{2}}}\right] = 0$

$\therefore$ No option is matching,

Question19

$\int \sqrt{4\cos^2 x - 5\sin^2 x} \cos x dx =$

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Options:

A. $\frac{1}{2}\cos x\sqrt{4 - 9\sin^2 x} + \frac{2}{3}\sin^{-1}\left(\frac{3\sin x}{2}\right) + c$

B. $\frac{1}{2}\sin x\sqrt{4 - 9\sin^2 x} + \frac{2}{3}\cos^{-1}\left(\frac{3\cos x}{2}\right) + c$

C. $\frac{1}{2}\cos x\sqrt{1 - 9\cos^2 x} + \frac{2}{3}\sin^{-1}\left(\frac{3\cos x}{2}\right) + c$

D. $\frac{1}{2}\sin x\sqrt{4 - 9\sin^2 x} + \frac{2}{3}\sin^{-1}\left(\frac{3\sin x}{2}\right) + c$

Answer: D

Solution:

Given,

$$I=\int \sqrt{4 \cos ^2 x-5 \sin ^2 x} \cos x d x$$

$$=\int \sqrt{4-9 \sin ^2 x} \cos x d x$$

Let $\sin x=t \Rightarrow \cos x d x=d t$

$$I=\int \sqrt{4-9 t^2} d t=3 \int \sqrt{\left(\frac{2}{3}\right)^2-t^2 d t}$$

$$\frac{1}{2} \sin x \sqrt{4-9 \sin ^2 x}+\frac{2}{2} \sin ^{-1}\left(\frac{3 \sin x}{2}\right)+C$$

Question20

$$\int\left(\frac{4 \tan ^4 x+3 \tan ^2 x-1}{\tan ^2 x+4}\right) d x=$$

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Options:

A. $4 \tan x-\frac{17}{4} \tan ^{-1}\left(\frac{\tan x}{4}\right)+c$

B. $4 \tan x-\frac{17}{4} \tan ^{-1}\left(\frac{\tan x}{2}\right)+c$

C. $4 \tan x-\frac{17}{2} \tan ^{-1}\left(\frac{\tan x}{2}\right)+c$

D. $2 \tan x-\frac{17}{2} \tan ^{-1}\left(\frac{\tan x}{2}\right)+c$

Answer: C

Solution:

Given,

$$I=\int\left(\frac{4 \tan ^4 x+3 \tan ^2 x-1}{\tan ^2 x+4}\right) d x$$

$$=\int\left(\frac{4 \tan ^4 x+4 \tan ^3 x-\tan ^2 x-1}{\tan ^2 x+4}\right) d x$$

$= \int \frac{(4\tan^2 x - 1)(\tan^2 x + 1)}{(\tan^2 x + 4)} dx$

Also, let $\tan x = t \Rightarrow \sec^2 x dx = dt$

$\Rightarrow (1 + \tan^2 x) dx = dt$

$\therefore \quad I = \int \frac{(4t^2 - 1)}{t^2 + 4} dt$

$I = 4 \int \left[\frac{(t^2+4)}{(t^2+4)} - \frac{17}{4} \times \frac{1}{(t^2+4)} \right] dt$

$I = 4 \int \left(1 - \frac{17}{4(t^2+4)} \right) dt$

$I = 4 \left[t - \frac{17}{4} \times \frac{1}{2} \tan^{-1} \left(\frac{t}{2} \right) + C \right]$

$I = 4 \tan x - \frac{17}{2} \tan^{-1} \left(\frac{\tan x}{2} \right) + C$

Question21

$$\int \left(\frac{(\sin^4 x + 2\cos^2 x - 1) \cos x}{(1 + \sin x)^6} \right) dx =$$

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Options:

A. $\frac{\sin^6 x}{6(1+\sin x)^6} + c$

B. $-\frac{\sin^6 x}{6(1+\sin x)^6} + c$

C. $\frac{\cos^6 x}{6(1+\sin x)^6} + c$

D. $-\frac{\cos^6 x}{6(1+\sin x)^6} + c$

Answer: A

Solution:

Given, $\int \frac{(\sin^4 x + 2\cos^2 x - 1) \cos x}{(1+\sin x)^6} dx$

Let $\sin x = t \Rightarrow \cos x dx = dt$

then, $\int \frac{\left[t^4+2\left(1-t^2\right)-1\right]}{(1+t)^6} dt$

$\because \quad t^4+2-2t^2-1$

$\Rightarrow \quad t^4-2t^2+1=\left(t^2-1\right)^2$

$\int \frac{\left(t^2-1\right)^2}{(1+t)^6} dt$

$\int \frac{(t-1)^2(t+1)^2}{(1+t)^6} dt$

$\int \frac{(t-1)^2}{(1+t)^4} dt$

Let $1+t=\mu, dt=d\mu$

$\int \frac{(\mu-1-1)^2}{\mu^4} d\mu$

$\int\left(\frac{\mu^2}{\mu^4}+\frac{4}{\mu^4}-\frac{4\mu}{\mu^4}\right) d\mu$

$\int \frac{1}{\mu^2} d\mu+4 \int \frac{1}{\mu^4} d\mu-4 \int \frac{1}{\mu^3} d\mu$

$=\frac{\mu^{-2+1}}{-2+1}+\frac{4\mu^{-4}}{-4+1}-\frac{4\mu^{-3+1}}{-3+1}+C$

$=\frac{-1}{\mu}-\frac{4}{3\mu^3}+\frac{2}{\mu^2}+C$

$=\frac{-3\mu^2-4+6\mu}{3\mu^3}+C$

Put $x=1+\sin x$

$(\because t=\sin x)$

$=\frac{-3(1+\sin x)^2-4+6(1+\sin x)}{3(1+\sin x)^3}+C$

$=\frac{-3-3\sin^2 x-6\sin x-4+6+6\sin x}{3(1+\sin x)^3}+C$

$=\frac{-3\sin^2 x-1}{3(1+\sin x)^3}+C=\frac{-\left(3\sin^2 x+1\right)}{3(\sin x+1)^2}+C$

Question22

$\int(\log x)^3 dx=$

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Options:

A. $(\log x)^3 - 3(\log x)^2 + 6\log x - 6 + c$

B. $x\left[(\log x)^3 - 3(\log x)^2 + 6(\log x) - 6\right] + c$

C. $(x\log x)^3 - 3x(\log x)^2 + 6x(\log x) - 6 + c$

D. $\frac{1}{x}\left[(\log x)^3 - 3(\log x)^2 + 6\log x - 6\right] + c$

Answer: B

Solution:

Using,

$\int u \cdot v dx = u \cdot \int v dx - \int \left(du \times \int v dx\right) dx$

Let $u = (\log x)^3$ and $v = 1$

$I = (\log x)^3 \cdot \int 1 dx - \int \left[\frac{d}{dx}(\log x)^3\right] \times \int 1 dx\big) dx$

$= x(\log x)^3 - \int \left(3(\log x)^2 \times \frac{1}{x} \times x\right) dx$

$= x(\log x)^3 - 3\int (\log x)^2 dx$

$= x(\log x)^3 - 3\left[(\log x)^2 \int 1 dx\right.$

$\left. - \int \left((\log x)^2 \times \int 1 dx\right) dx\right]$

$= x(\log x)^3 - 3 \quad \left[x(\log x)^2 - \int \left(2(\log x) \times \frac{1}{x} \times x\right) dx\right]$

$= x(\log x)^3 - 3x(\log x)^2 + 6\int \log x dx$

$= x(\log x)^3 - 3x(\log x)^2 + 6[\log x \times$

$\int 1 dx - \int \left((\log x) \times \int 1 dx\right) dx]$

$= x(\log x)^3 - 3x(\log x)^2 + 6\left[x\log x - \int 1 dx\right]$

$= x\left[(\log x)^3 - 3(\log x)^2 + 6\log x - 6\right] + c$

Question23

If $\frac{3x^4-2x^2+1}{(x-2)^4} = A + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3} + \frac{E}{(x-2)^4}$**, then** $2A + 3B - C - D + E =$

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Options:

A. 0

B. 1

C. -11

D. -39

Answer: D

Solution:

$$\text{Given, } \frac{3x^4-2x^2+1}{(x-2)^4}$$

$$= A + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3} + \frac{E}{(x-2)^4}$$

$$\Rightarrow 3x^4 - 2x^2 + 1 = A(x-2)^4$$

$$+B(x-2)^3 + C(x-2)^2 + D(x-2) + E$$

$$\Rightarrow 3x^4 - 2x^2 + 1 = Ax^4 + (-8A+B)x^3$$

$$+(24A - 6B + C)x^2$$

$$+(-32A + 12B - 4C + D)x$$

$$+(16A - 8B + 4C - 2D + E)$$

On comparing coefficients, we get

$$A = 3, -8A + B = 0$$

$$\Rightarrow \quad B = 24$$

$$24A - 6B + C = -2$$

$$\Rightarrow \quad C = 70, -32A + 12B - 4C + D = 0$$

$$\Rightarrow \quad D = 88$$

$$16A - 8B + 4C - 2D + E = 1$$

$$\Rightarrow \quad E = 41$$

$$\therefore \quad 2A + 3B - C - D + E$$

$$= 2(3) + 3(24) - 70 - 88 + 41$$

$$= (6 + 72 + 41) - (70 + 88)$$

$$= 119 - 158 = -39$$

Question24

$\int e^{-2x}\left(\tan 2x - 2\sec^2 2x \tan 2x\right) dx =$

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Options:

A. $e^{-2x} \tan 2x + c$

B. $-\frac{e^{-2x}}{2}\left[\sec^2 2x + \tan 2x\right] + c$

C. $-\frac{e^{-2x}}{2}\left[\tan 2x - \sec^2 2x\right] + c$

D. $e^{-2x} \sec^2 2x + c$

Answer: B

Solution:

Let

$$I=\int e^{-2x}\left(\tan 2x-2\sec^2 2x\tan 2x\right)dx$$

Put $-2x=t$

$$\Rightarrow \quad dx=-\frac{dt}{2}$$

$$\therefore I=\int e^t\left(\tan(-t)-2\sec^2(-t)\tan(-t)\right)\left(-\tfrac{dt}{2}\right)$$

$$=-\frac{1}{2}\int e^t\left(2\sec^2 t\tan t-\tan t\right)dt$$

$$=-\frac{1}{2}\int e^t\left[\left(\sec^2 t-\tan t\right)\right.$$

$$\left.+\left(2\sec^2 t\tan t-\sec^2 t\right)\right]dt$$

We know that,

$$\int e^t f(t)+f'(t)dt=e^t f(t)+C$$

Here, $\quad f(t)=\sec^2 t-\tan t$

and $\quad f'(t)=2\sec^2 t\tan t-\sec^2 t$

$$\therefore \quad I=-\frac{1}{2}e^t\left[\sec^2 t-\tan t\right]+C$$

$$=-\frac{1}{2}e^{-2x}\cdot\left[\sec^2(-2x)-\tan(-2x)\right]+C$$

$$=-\tfrac{e^{-2x}}{2}\left[\sec^2 2x+\tan 2x\right]+c$$

Question25

If $\int x^3\sin 3x dx=f(x)\cos 3x+g(x)\sin 3x+C$, then 27 $(f(x)+xg(x))=$

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Options:

A. $18x^3+4x$

B. $8x$

C. $4x$

D. $18x^3 + 8x$

Answer: C

Solution:

Given, $\int x^3 \sin 3x dx$

$$= f(x)\cos 3x + g(x)\sin 3x + C$$

Let, $I = \int x^3 \sin 3x dx$

$$= x^3 \int \sin 3x dx - \int \left(3x^2 x \int \sin 3x dx\right) dx$$

$$= x^3\left(\frac{-\cos 3x}{3}\right) - \int \left(3x^2 \times \left(\frac{-\cos 3x}{3}\right)\right) dx$$

$$= -\frac{x^3}{3}\cos 3x + \int x^2 \cos 3x dx$$

$$= -\frac{x^3}{3}\cos 3x + \left[x^2\left(\frac{\sin 3x}{3}\right) - \int \left(2x \times \frac{\sin 3x}{3}\right) dx\right]$$

$$= -\frac{x^3}{3}\cos 3x + \frac{x^2}{3}\sin 3x - \frac{2}{3}\int x \sin 3x dx$$

$$= -\frac{x^3}{3}\cos 3x + \frac{x^2}{3}\sin 3x - \frac{2}{3}$$

$$\left[\frac{-x}{3}\cos 3x + \frac{\sin 3x}{9}\right] + C$$

$$= \left(-\frac{x^3}{3} + \frac{2x}{9}\right)\cos 3x + \left(\frac{x^2}{3} - \frac{2}{27}\right)\sin 3x + 6$$

$$\therefore f(x) = -\frac{x^3}{3} + \frac{2x}{9} \text{ and } g(x) = \frac{x^2}{3} - \frac{2}{27}$$

Now, $27(f(x)+xg(x))$

$$=27\left[\left(-\frac{x^3}{3}+\frac{2x}{9}\right)+x\left(\frac{x^2}{3}-\frac{2}{27}\right)\right]$$

$$=27\left[\frac{2x}{9}-\frac{2}{27}x\right]=6x-2x=4x$$

Question26

$$\int \frac{dx}{9\cos^2 2x+16\sin^2 2x}=$$

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Options:

A. $\frac{1}{25}\tan^{-1}\left(\frac{3}{4}\sec^2 2x\right)+c$

B. $\frac{1}{25}\tan^{-1}\left(\frac{4}{3}\sec^2 2x\right)+c$

C. $\frac{1}{24}\tan^{-1}\left(\frac{3}{4}\tan 2x\right)+c$

D. $\frac{1}{24}\tan^{-1}\left(\frac{4}{3}\tan 2x\right)+c$

Answer: D

Solution:

$$\text{Let } I=\int \frac{dx}{9\cos^2 2x+16\sin^2 2x}$$

$$=\int \frac{\sec^2 2x dx}{9+16\tan^2 2x}=\frac{1}{9}\int \frac{\sec^2 2x dx}{1+\left(\frac{4}{3}\tan 2x\right)^2}$$

$$=\frac{1}{24}\int \frac{\frac{4}{3}\cdot\left(2\sec^2 2x\right)}{1+\left(\frac{4}{3}\tan 2x\right)^2}$$

$$= \frac{1}{24}\tan^{-1}\left(\frac{4}{3}\tan 2x\right) + C$$

$$\left[\because \int \frac{f'(x)dx}{1+[f(x)]^2} = \tan^{-1}(f(x)) + C\right]$$

Question27

$\int \frac{2\cos 3x - 3\sin 3x}{\cos 3x + 2\sin 3x} dx =$

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Options:

A. $\frac{7}{15}\log|\cos 3x + 2\sin 3x| - \frac{4}{5}x + c$

B. $-\frac{4}{5}\log|\cos 3x + 2\sin 3x| + \frac{7x}{5} + c$

C. $\frac{7}{5}\log|\cos 3x + 2\sin 3x| - \frac{4}{5}x + c$

D. $-\frac{8}{15}\log|\cos 3x + 2\sin 3x| + \frac{x}{5} + c$

Answer: A

Solution:

Let,

$I = \int \frac{2\cos 3x - 3\sin 3x}{\cos 3x + 2\sin 3x} dx$

Put $3x = t \Rightarrow 3dx = dt$

$\therefore \quad I = \int \frac{2\cos t - 3\sin t}{\cos t + 2\sin t}\left(\frac{dt}{3}\right)$

Let, $2\cos t - 3\sin t = A(\cos t + 2\sin t)$

$+B\left[\frac{d}{dt}(\cos t + 2\sin t)\right]$

$\Rightarrow 2\cos t - 3\sin t = A(\cos t + 2\sin t)$

$+B(-\sin t + 2\cos t)$

$\Rightarrow 2 = A + 2B$ and $-3 = 2A - B$

On solving these equations, we get

$A = -\frac{4}{3}$ and $B = \frac{7}{5}$

$\therefore I=\frac{1}{3}\left[\int \frac{-\frac{4}{5}(\cos t+2 \sin t)}{+\frac{7}{5}(-\sin t+2 \cos t)}(\cos t+2 \sin t] d t\right]$

$$=\frac{1}{3}\left[-\frac{4}{5} t+\frac{7}{5} \log |\cos t+2 \sin t|\right]+C$$

$$=\frac{7}{15} \log |\cos 3 x+2 \sin 3 x|-\frac{4}{5} x+C$$

$[\because t=3 x]$

Question28

If $\frac{x^4}{(x^2+1)(x-2)}=f(x)+\frac{A x+B}{x^2+1}+\frac{C}{x-2}$, then $f(14)+2 A-B=$

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Options:

A. $5 C$

B. $4 C$

C. $6 C$

D. $7 C$

Answer: A

Solution:

We have,

$$\frac{x^4}{(x^2+1)(x-2)}=f(x)+\frac{A x+B}{x^2+1}+\frac{C}{x-2}$$

$$\frac{x^4}{(x^2+1)(x-2)}$$

$$f(x)\left[\left(x^2+1\right)(x-2)\right]$$

$$= \frac{+(Ax+B)(x-2)+C\left(x^2+1\right)}{(x^2+1)(x-2)}$$

$$\Rightarrow \frac{x^4}{(x^2+1)(x-2)}$$

$$f(x)\left[x^3-2x^2+x-2\right]$$

$$= \frac{+\left[Ax^2+(B-2A)x-2B\right]+Cx^2+C}{(x^2+1)(x-2)}$$

Here, $f(x)$ will be linear function only as highest power in LHS is 4

So, let $f(x)=px+q$

$\frac{x^4}{(x^2+1)(x-2)}$

$$(px+q)\left[x^3-2x^2+x-2\right]$$

$$= \frac{+\left[Ax^2+(B-2A)x-2B+cx^2+C\right]}{(x^2+1)(x-2)}$$

$$\Rightarrow \frac{x^4}{(x^2+1)(x-2)}$$

$$px^4+(q-2p)x^3+(p-2q+A+C)x^2$$

$$= \frac{+(q-2p+B-2A)x+(C-2B-2q)}{(x^2+1)(x-2)}$$

On comparing both sides, we get

$$p=1, q-2p=0 \Rightarrow q=2$$

$$p-2q+A+C=0 \Rightarrow A+C=3$$

$$q-2p+B-2A=0 \Rightarrow B=2A$$

$$C-2B-2q=0 \Rightarrow C-2B=4$$

From Eqs. (i), (ii) and (iii), we get,

$A=\frac{-1}{5}, C=\frac{16}{5}, B=\frac{-2}{5}, f(x)=x+2$

So, $f(14)+2A-B=14+2+0=16=5C$

Question29

$\int(\sqrt{1-\sin x}+\sqrt{1+\sin x})dx=f(x)+c$, where c is the constant of integration. If $\frac{5\pi}{2}<x<\frac{7\pi}{2}$ and $f\left(\frac{8\pi}{3}\right)=-2,\ \text{then}\ f'\left(\frac{8\pi}{3}\right)=$

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Options:

A. 1

B. $\sqrt{3}$

C. 0

D. -1

Answer: B

Solution:

We have,

$\int(\sqrt{1-\sin x}+\sqrt{1+\sin x})dx=f(x)+C$

Let

$I=\int(\sqrt{1-\sin x}+\sqrt{1+\sin x})dx$

$=\int\left[\left|\sin\frac{x}{2}-\cos\frac{x}{2}\right|+\left|\sin\frac{x}{2}+\cos\frac{x}{2}\right|\right]dx$

$=\int\left[\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)-\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)\right]dx$

$\left[\because \frac{5\pi}{2}<x<\frac{7\pi}{2}\Rightarrow\frac{5\pi}{4}<\frac{x}{2}<\frac{7\pi}{4}\right]$

$=\int\left(\cos\frac{x}{2}-\sin\frac{x}{2}-\sin\frac{x}{2}-\cos\frac{x}{2}\right)dx$

$=-2\int\sin\frac{x}{2}dx=4\cos\frac{x}{2}+C$

So, $f(x)=4\cos\frac{x}{2} \Rightarrow f'(x)=-2\sin\frac{x}{2}$

$f'\left(\frac{8\pi}{3}\right)=-2\sin\frac{4\pi}{3}=-2\times\frac{-\sqrt{3}}{2}=\sqrt{3}$

Question30

$$\int\frac{(1-4\sin^2 x)\cos x}{\cos(3x+2)}dx=$$

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Options:

A. $(\cos 2)x-\frac{1}{3}(\sin 2)\log|\sec(3x+2)|+c$

B. $(\sin 2)x-\frac{1}{3}(\cos 2)\log|\cos(3x+2)|+c$

C. $(\sin 2)x+\frac{1}{3}(\cos 2)\log|\cos(3x+2)|+c$

D. $(\cos 2)x+\frac{1}{3}(\sin 2)\log|\sec(3x+2)|+c$

Answer: D

Solution:

$$=\int\frac{(1-4\sin^2 x)\cos x}{\cos(3x+2)}dx$$

Let

$$=\int\frac{(1-4+4\cos^2 x)\cos x}{\cos(3x+2)}dx$$

$=\int\frac{4\cos^2 x-3\cos x}{\cos(3x+2)}dx=\int\frac{\cos 3x}{\cos(3x+2)}dx$

Let $3x+2=t \Rightarrow xdx=\frac{dt}{3}$

$$I=\int\frac{\cos(t-2)}{\cos t}\frac{dt}{3}$$

$$=\frac{1}{3}\int\frac{\cos 2\cos t+\sin 2\sin t}{\cos(t)}dt$$

$$= \frac{1}{3}\int(\cos 2 + \sin 2 \tan t)dt$$

$$= \frac{1}{3}[(\cos 2)\cdot t + \sin 2 \ln(\sec t) + K]$$

$$= \frac{1}{3}[\cos 2 \cdot (3x+2) + \sin 2 \cdot \ln[\sec(3x+2)] + K$$

$$=(\cos 2)x + \frac{2}{3}\cos 2 + \frac{1}{3}\sin 2 \cdot \ln[\sec(3x+2)] + \frac{K}{3}$$

$$= (\cos 2)x + \tfrac{1}{3}\sin 2 \ln[\sec(3x+2)] + C$$

Question31

$\int \frac{(1-4\sin^2 x)\cos x}{\cos(3x+2)} dx =$

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Options:

A. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^4-1}{\sqrt{2}x^2}\right)+c$

B. $\log\left(x^5+x^2\right)-\log\left(x^3+x\right)+\log(x+1)+c$

C. $\frac{2}{9}x^8-\frac{4}{9}x^6+\frac{1}{9}x^4-\frac{1}{3}x^2+c$

D. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^5-1}{\sqrt{2}x^3}\right)+c$

Answer: A

Solution:

Let $I = \int \frac{x^5+x}{x^8+1}dx$

$$= \int \frac{x^4\left(x+\frac{1}{x^3}\right)}{x^4\left(x^4+\frac{1}{x^4}\right)}dx = \int \frac{\left(x+\frac{1}{x^3}\right)dx}{(x^2)^2+\left(\frac{1}{x^2}\right)^2-2+2}$$

$$= \int \frac{\left(x+\frac{1}{x^3}\right) dx}{\left(x^2-\frac{1}{x^2}\right)^2+(\sqrt{2})^2}$$

Let $x^2-\frac{1}{x^2}=t \Rightarrow 2\left(x+\frac{1}{x^3}\right) dx=dt$

So, $I=\int \frac{dt/2}{t^2+(\sqrt{2})^2}=\frac{1}{2} \int \frac{dt}{t^2+\sqrt{2}}=\frac{1}{2}\left[\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{t}{\sqrt{2}}\right)\right]=\frac{1}{2\sqrt{2}} \tan^{-1}\left[\frac{x^4-1}{\sqrt{2} x^2}\right]+c$

[put the value of $t=x^2-\frac{1}{x^2}$]

Question32

If $\frac{x+1}{(x^2+1)(x-1)^2}=\frac{Ax+B}{x^2+1}+\frac{C}{x-1}+\frac{D}{(x-1)^2}$**, then** $A+B+C+D=$

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Options:

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. 1

D. $\frac{3}{2}$

Answer: B

Solution:

Given, $\frac{x+1}{(x^2+1)(x-1)^2}=\frac{(Ax+B)}{x^2+1}+\frac{C}{x-1}+\frac{D}{(x-1)^2}$

$\Rightarrow \frac{x+1}{(x^2+1)^2(x-1)^2}$

$=\frac{(Ax+B)(x-1)^2+C(x-1)\left(x^2+1\right)+D\left(x^2+1\right)}{(x^2+1)(x-1)^2}$

$\Rightarrow (x+1)=(Ax+B)(x-1)^2+C(x-1)\left(x^2+1\right)+D\left(x^2+1\right)$

Put $x=0$,

$$1 = B(-1)^2 + C(-1) + D$$
$$1 = B - C + D \qquad \ldots(i)$$

Put $x = 1$,

$$2 = D(2) \Rightarrow D = 1 \qquad \ldots(ii)$$

Put $x = -1$,

$$0 = 4(B - A) + C(-2)(2) + D(2)$$
$$\Rightarrow 0 = 4B - 4A - 4C + 2D$$
$$\Rightarrow 2B - 2A - 2C + D = 0$$
$$\Rightarrow 2B - 2A - 2C = -1$$
$$\Rightarrow 2(B - C) - 2A = -1 \quad [\because (B - C) = 0]$$
$$\Rightarrow 0 - 2A = -1$$
$$\Rightarrow A = \frac{1}{2}$$

Put $x = 2$,

$$3 = (2A + B) + C(5) + D(5)$$
$$\Rightarrow 3 = 2 \times \frac{1}{2} + B + 5C + 5$$
$$\Rightarrow B + 5C = -3 \text{ and } B - C = 0$$

Now, $B = -\frac{3}{6} = -\frac{1}{2}$ and $C = -\frac{1}{2}$

Now,

$$A + B + C + D = \frac{1}{2} + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right) + 1$$
$$= \frac{1}{2}$$

Question33

Match the following items from List I into List II

List-I	List-II
1. $\int \frac{\sin^2 x}{\cos^4 x} dx$	A. $\frac{\tan^2 x}{2} + \ln\lvert\cos x\rvert + C$
2. $\int \frac{\sin^4 x}{\cos^2 x} dx$	B. $\cos x + \sec x + C$
3. $\int \frac{\sin^3 x}{\cos^2 x} dx$	C. $\frac{\tan^3 x}{3} + C$
4. $\int \frac{\sin^3 x}{\cos^3 x} dx$	D. $\tan x + \frac{\sin 2x}{4} - \frac{3x}{2} + C$
	E. $\cos x - \sec x + C$

Select the correct choice

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Options:

A. 1-C, 2-E, 3-B, 4-A

B. 1,-C, 2-D, 3-B, 4-A

C. 1-D, 2-C, 3-A, 4-B

D. 1-C, 2-E, 3-A, 4-D

Answer: B

Solution:

Now,

$$\int \frac{\sin^2 x}{\cos^4 x} dx = \int \tan^2 x \sec^2 x dx$$
$$= \frac{\tan^3 x}{3} + C$$

So, $(1) \rightarrow (C)$

Also, $\int \frac{\sin^4 x}{\cos^2 x} dx = \int \frac{\sin^2 x(1-\cos^2 x)}{\cos^2 x} dx$

$$= \int \left(\tan^2 x - \sin^2 x\right) dx$$
$$= \int \left(\sec^2 x - 1 - \frac{1}{2}(1 - \cos 2x)\right) dx$$
$$= \int \sec^2 x dx + \frac{1}{2} \int \cos 2x dx - \frac{3}{2} \int dx$$
$$= \tan x + \frac{1}{4} \sin 2x - \frac{3}{2} x + C$$

$(2) \rightarrow (D)$

$$For\ (3) \int \frac{\sin^3 x}{\cos^2 x} dx$$
$$= \int \frac{\sin x \left(1 - \cos^2 x\right)}{\cos^2 x} dx$$
$$= \int \sec x \tan x dx - \int \sin x dx$$
$$= \sec x + \cos x + C$$

(3) $\rightarrow$ (B)

$For\ (4)\int \frac{\sin^3 x}{\cos^3 x} dx = \int \tan^3 x dx$

$= \int \tan x \sec^2 x dx - \int \tan x dx$

$= \frac{\tan^2 x}{2} + \log|\cos x| + C$

$(4) \rightarrow (A)$

Question34

If $\int \frac{x}{(a+x)^5} dx = \frac{1}{k(a+x)^4}(f(x)) + C$, then $\frac{f(-a)}{ak} =$

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Options:

A. $1/3$

B. $1/2$

C. $5/6$

D. $1/4$

Answer: D

Solution:

Given,

$$\int \frac{x}{(a+x)^5} dx = \frac{1}{k(a+x)^4}(f(x)) + C$$

$$\text{We have, LHS } = \int \frac{x}{(a+x)^5} dx$$

$$\text{or, } I = \int \frac{a+x-a}{(a+x)^5} dx$$

$$= \int \frac{a+x}{(a+x)^5} dx - \int \frac{a}{(a+x)^5} dx$$

$$= \int \frac{1}{(a+x)^4} dx - \int \frac{a}{(a+x)^5} dx$$

$$= \frac{-1}{3} \frac{1}{(a+x)^3} + \frac{a}{4} \frac{1}{(a+x)^4} + C$$

$$= \frac{-4x-a}{12(a+x)^4} + C$$

On comparing it with given equation, we get

$f(x) = -4x - a$ and $k = 12$

Then,

$$\begin{aligned} \frac{f(-a)}{ak} &= \frac{4a - a}{a \times 12} \\ &= \frac{3a}{12a} = \frac{1}{4} \end{aligned}$$

Question35

If $\int x^4(\log x)^3 dx = x^5 \left[A(\log x)^3\right] + B(\log x)^2 + C \log x + D\right] + k$, then $A + B + C + 5D =$

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Options:

A. $\frac{2}{25}$

B. $\frac{8}{25}$

C. $\frac{12}{125}$

D. $\frac{16}{125}$

Answer: A

Solution:

$\int x^4(\log x)^3 dx$

$$\begin{aligned} &= (\log x)^3 \frac{x^5}{5} - \int 3(\log x)^2 \cdot \frac{1}{x} \cdot \frac{x^5}{5} dx \\ &= \frac{x^5}{5}(\log x)^3 - \frac{3}{5} \int x^4(\log x)^2 dx \\ &= \frac{x^5}{5}(\log x)^3 - \frac{3}{5}\left[(\log x)^2 \frac{x^5}{5} \quad - \int 2(\log x) \cdot \frac{1}{x} \cdot \frac{x^5}{5} dx\right] \end{aligned}$$

$$=\frac{x^5}{5}(\log x)^3-\frac{3}{25}(\log x)^2x^5 \quad +\frac{6}{25}\int x^4(\log x)dx$$

$$=\frac{x^5}{5}(\log x)^3-\frac{3}{25}(\log x)^2x^5+\frac{6}{25} \quad \left[\frac{x^5}{5}(\log x)-\int\frac{1}{x}\cdot\frac{x^5}{5}dx\right]$$

$$=\frac{x^5}{5}(\log x)^3-\frac{3}{25}(\log x)^2x^5 \quad +\frac{6}{125}(\log x)x^5-\frac{6}{125}\frac{x^5}{5}+K$$

$$=x^5\left\{\frac{1}{5}(\log x)^3-\frac{3}{25}(\log x)^2 \quad +\frac{6}{125}(\log x)-\frac{6}{625}\right\}+C$$

Thus, $A=\frac{1}{5}, B=-\frac{3}{25}, C=\frac{6}{125}$ and

$D=-\frac{6}{625}$

$\therefore A+B+C+5D$

$$=\frac{1}{5}-\frac{3}{25}+\frac{6}{125}-\frac{6}{125}=\frac{2}{25}$$

Question36

If $\frac{x^4}{(x-1)(x-2)(x-3)}=p(x)+\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3}$, then $p\left(\frac{3}{2}\right)+C=$

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Options:

A. 0

B. 8

C. $-17/2$

D. 48

Answer: D

Solution:

$$(x-1)(x-2)(x-3)$$
$$=x^3-6x^2+11x-6$$

$$x^3-6x^2+11x-6)x^4$$
$$\frac{x^4-6x^3+11x^2-6x+6}{6x^3-11x^2+6x}$$
$$\frac{6x^3-36x^2+66x-36}{(-)(+)(-)}$$
$$25x^2-60x+36$$
$$\therefore x^4=(x+6)(x-1)(x-2)(x-3)$$
$$\Rightarrow \frac{25x^2-60x+36}{(x-1)(x-2)(x-3)}$$
$$(x-1)(x-2)(x-3)$$
$$x^4(x+6)$$
$$p(x)=(x+6)$$
$$\text{and } \frac{25x^2-60x+1/6}{(x-1)(x-2)(x-3)}$$
$$=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3}$$
$$\Rightarrow C=\frac{25\left(3^2\right)-60\times 3+36}{(3-1)(3-2)}=\frac{81}{2}$$
$$\therefore p\left(\frac{3}{2}\right)+C=\left(\frac{3}{2}+6\right)+\frac{81}{2}=48$$

Question37

$$\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x} dx=$$

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Options:

A. $\frac{1}{2}\cos 2x+c$

B. $\frac{-1}{2}\cos 2x+c$

C. $\frac{-1}{(1+\tan x)^2}+C$

D. $\frac{-1}{2}\sin 2x+C$

Answer: D

Solution:

(d) $I=\int\left(\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}\right)dx$

$$I=\int\frac{\left(\sin^4 x-\cos^4 x\right)\left(\sin^4 x+\cos^4 x\right)}{\left(1-2\sin^2 x\cos^2 x\right)}$$

$$\left(\sin^2 x-\cos^2 x\right)\left(\sin^2 x+\cos^2 x\right)$$

$$I=\int\frac{\left\{\left(\sin^2 x+\cos^2 x\right)^2-2\sin^2 x\cos^2 x\right\}}{1-2\sin^2 x\cos^2 x}dx$$

$$I=\int\frac{(-\cos 2x)(1)\left(1-2\sin^2 x\cos^2 x\right)}{1-2\sin^2 x\cos^2 x}dx$$

$$I=-\frac{\sin(2x)}{2}+C$$

Question38

If $\int\frac{x^2(\sec^2 x+\tan x)}{(x\tan x+1)^2}dx=\frac{-x^2}{x\tan x+1}+f(x)+C$, **then** $f(x)=$

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Options:

A. $\log|x\sin x+\cos x|+C$

B. $\log|x\cos x+\sin x|+C$

C. $2\log|x\sin x+\cos x|+C$

D. $2\log|x\cos x+\sin x|+C$

Answer: C

Solution:

$$\int\frac{x^2\left(\sec^2 x+\tan x\right)}{(x\tan x+1)^2}dx$$

$$=\int\frac{x^2\sec^2 x+x^2\tan x}{(x\tan x+1)^2}dx$$

$$=\int\left[\frac{-x}{x\tan x+1}+\frac{x\left(x\sec^2 x-1\right)}{(x\tan x+1)^2}\right.$$

$$+\frac{2x^2\tan x+2x}{(x\tan x+1)^2}\Bigg]dx$$

$$I=\int\left[\frac{-x}{x\tan x+1}+x\frac{x\sec^2 x-1}{(x\tan x+1)^2}\right]dx$$

$$I_1$$

$$+\int\frac{2x^2\tan x+2x}{(x\tan x+1)^2}dx$$

$$I_2$$

$$I_1=\int\left[\frac{-x}{x\tan x+1}+x\frac{xI_1+I_2}{(x\sec^2 x-1}\right]dx$$

$$\because\int\big(f(x)+xf'(x)\big)dx=xf(x)+C$$

$$\therefore I_1=\frac{-x^2}{x\tan x+1}$$

$$I_2=\int\frac{2x^2\tan x+2x}{(x\tan x+1)^2}dx$$

$$=\int\frac{2x}{x\tan x+1}dx$$

$$=\int\frac{2x\cos x}{x\sin x+\cos x}dx$$

Let $x\sin x+\cos x=t$

$$\therefore\quad I_2=\int\frac{2dt}{t}=2\ln t$$

$$I_2=2\ln|x\sin x+\cos x|$$

$$I=\frac{-x^2}{x\tan x+1}+2\ln|x\sin x+\cos x|+C$$

Question39

If $\int\sin(101x)(\sin x)^{99}dx=\frac{\sin(100x)(\sin x)^{\lambda}}{\mu}+C$ **then,** $\frac{\lambda}{\mu}=$

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Options:

A.

1

B.

2

C.

4

D.

8

Answer: A

Solution:

$$I=\int \sin(101x)(\sin x)^{99}dx$$
$$=\int \sin(100x+x)(\sin x)^{99}dx$$
$$=\int \sin(100x)\cos x(\sin x)^{99}dx+$$
$$\int \cos(100x)\sin^{100}xdx$$
$$=\int \cos x(\sin x)^{99}\sin 100xdx+I_1$$
$$I_1=\int \cos(100x)\sin^{100}xdx$$
$$=\sin^{100}x\int \cos 100xdx-$$
$$\int\left[\left(\int \cos 100xdx\right)\left(\frac{d\sin^{100}x}{dx}\right)\right]dx$$
$$=\frac{\sin^{100}x\sin 100x}{100}-$$
$$\int\left(\frac{\sin 100x}{100}\cdot 100(\sin x)^{99}\cos x\right)dx$$
$$I=\int \cos x(\sin x)^{99}\sin 100xdx$$
$$+\frac{\sin^{100}x\sin 100x}{100}-\int \cos x(\sin x)^{99}$$
$$\sin 100x\times dx+C$$
$$I=\int\frac{\sin^{100}x\sin(100x)}{100}+C$$
$$\lambda=100,\mu=100$$
$$\therefore \frac{\lambda}{\mu}=1$$

Question40

If $\int e^x \left(\sin^2 2x - 8\cos 4x\right) dx = e^x f(x) + C$, then $f\left(\frac{\pi}{4}\right) =$

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Options:

A. 0

B. 1

C. -1

D. e

Answer: B

Solution:

$$\int e^x \left(\sin^2 2x - 8\cos 4x\right) dx$$
$$= \int e^x \left(\frac{1-\cos 4x}{2} + 2\sin 4x\right.$$
$$\left. - 2\sin 4x - 8\cos 4x\right) dx$$
$$= \int e^x \left(\frac{1-\cos 4x}{2} + 2\sin 4x\right) dx$$
$$- 2\int e^x (\sin 4x + 4\cos 4x) dx$$
$$= e^x \left(\frac{1-\cos 4x}{2}\right) - 2e^x \sin 4x + C$$
$$= e^x \left(\frac{1-\cos 4x}{2} - 2\sin 4x\right) + C$$
$$\Rightarrow f(x) = \frac{1-\cos 4x}{2} - 2\sin 4x$$
$$\therefore f\left(\frac{\pi}{4}\right) = \frac{1+1}{2} - 0 = 1$$

Question41

If n is a positive integer greater than 1 and $I_n = \int \frac{\sin nx}{\sin x} dx$, then $I_{n+1} - I_{n-1} =$

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Options:

A. $\frac{2}{n-1} \cos (n-1) x$

B. $\frac{2}{n-1} \sin (n-1) x$

C. $\frac{2}{n} \cos n x$

D. $\frac{2}{n} \sin n x$

Answer: D

Solution:

$$\text{Given, } I_n=\int \frac{\sin n x}{\sin x} d x$$

$$I_{n+1}-I_{n-1}=\int\left(\frac{\sin (n+1) x}{\sin x}-\frac{\sin (n-1) x}{\sin x}\right) d x$$

$$=\int\left(\frac{\sin (n+1) x-\sin (n-1) x}{\sin x}\right) d x$$

$$=\int \frac{2 \sin x \cos n x}{\sin x} d x$$

$$=\int 2 \cos n x d x=\frac{2}{n} \sin n x$$
